Interference-Aware MAC Protocol for Wireless Networks by a Game-Theoretic Approach

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Abstract—We propose an interference-aware MAC protocol using a simple transmission strategy motivated by a game-theoretic approach. We formulate a channel access game, which considers nodes concurrently transmitting in nearby clusters, incorporating a realistic wireless communication model - the SNR model. Under inter-cluster interference, we derive a decentralized transmission strategy, which achieves a Bayesian Nash Equilibrium (BNE). The proposed MAC protocol balances network throughput and battery consumption at each transmission.

To mitigate this effect, several decentralized algorithms motivated by game theory have been proposed [8]–[15]. In centralized approaches, the network system cannot easily protect from user deviation (i.e., a unilateral bid to increase throughput), and so may result in an unbalanced outcome [16]. On the other hand, the Nash Equilibria found in these decentralized approaches characterize points where a selfish user cannot benefit by unilaterally deviating; such an approach is resilient to deviation and ensures the robustness of the network system. However, these works do not explicitly handle the effect of interference, and are unable to accurately model realistic channel environments as they do not consider concurrent transmissions. Some recent game-theoretic works [17], [18] do consider the effect of interference and obtain a complex NE-based power adaptation scheme to mitigate packet collision.

The key advantage of studying MAC protocol design from a game theoretic perspective is that, by turning nodes into selfish players, an otherwise complex system can reach efficient outcomes in a lightweight and distributed manner, while reducing the communication overhead between nodes and their home APs, and between neighboring APs. In this paper, we focus on the channel access problem under inter-cluster interference from nearby APs, and find a simple transmission strategy that determines whether to Transmit or Backoff depending on channel conditions. We focus on the case that each node experiences instantaneous signal-to-noise ratio (SNR) fluctuation caused by interferers concurrently transmitting. First, assuming that each transmitter knows the number of active users at the next transmission, its own channel gain, and the probability distribution of the gains of interfering channels, we derive a threshold-based transmission scheme balancing benefit (i.e., throughput) and cost (i.e., transmission cost) which achieves a Bayesian Nash Equilibrium. Then, we present a simple dynamic procedure for nodes to efficiently find a Nash Equilibrium without requiring nodes to know the total number of active nodes or even their channel gain distributions.

Our proposed MAC protocol does not assign any backoff slot beforehand for later access. Rather, after a random backoff time, given a time-varying channel state, a subsequent channel access is attempted. Our protocol is not restricted to a specific homogeneous network, and can be extended to heterogeneous networks such as 802.11x, 802.15.4 sensor networks and Bluetooth, by using each different crosstalk ratio.

To the best of our knowledge, this paper is the first to for-
mulate a channel access game under the effect of inter-cluster interference incorporating a realistic wireless communication model. Our main contributions can be summarized as:

- We formulate an interference-aware channel access game, which considers concurrent transmission of nodes from different APs, incorporating random fading channels and a realistic wireless communication model.
- We prove the existence of a Bayesian Nash Equilibrium of the game, and compute it using numerical methods.
- We compare our BNE-based decentralized strategy with a centralized globally optimal strategy in terms of efficiency and balance.
- We present a simple dynamic procedure for nodes to efficiently find a Nash Equilibrium without requiring each node to know the total number of active nodes or the channel gain distribution, and prove that this procedure is guaranteed to converge.

The rest of this paper is organized as follows: In Sec. II, we discuss related work. Sec. III formally defines our channel access problem. In Sec. IV, we motivate our use of game theory, and then formulate the problem as a channel access game. Sec. V describes the proposed static game-theoretic approach, and then we evaluate the approach in Sec. VI. We prove a convergence result using best-response dynamics to find a Nash Equilibrium in Sec. VII, and then conclude in Sec. VIII.

II. RELATED WORK

The channel access game problem has recently been studied in the wireless MAC research community, with game-theoretic MAC schemes taking their solutions from three different categories: NE-based backoff time adaptation, NE-based power control, and NE-based transmission schemes that depend on channel conditions.

Some previous works [19], [20] consider the exponential backoff scheme in the IEEE 802.11 standard, and formulate the backoff protocol as a non-cooperative game where each link tries to maximize its own utility. They prove the existence of a Nash Equilibrium, and show that the NE-based backoff scheme converges to a globally optimal point of network utility.

Adaptive power control schemes are proposed in [17], [18]. Both works formulate the problem as a power control game, where strategies are given by interference-dependent transmission power profiles. These works explicitly consider the interference effect as in our work, but their strategies provide schemes that optimize over power adaptation, whereas our work optimizes a transmission scheme given a constant power.

Some recent works are closely related to our approach. Qin and Berry [9] propose a channel-aware ALOHA protocol, and compute a user’s transmission probability given its own channel gain. They assume that a collision occurs when two or more users transmit packets in the same slot. In [12], [15], transmission strategies decide whether or not to access the channel, i.e., Transmit or Backoff, depending on the SNR. They assume that each user’s SNR is an independent and identically-distributed (i.i.d.) random variable, and follows the probability density function of a Rayleigh fading channel. However, when we consider nodes concurrently transmitting out of the home AP, i.e., taking into account the inter-cluster interference effect, their assumptions are no longer valid. This is because the SNR is not in fact i.i.d., but depends on the number of nodes concurrently transmitting, which cannot be known before the game ends. Our work relaxes these assumptions so that only each channel gain is assumed to be i.i.d., and the SNR is expressed in terms of the channel gains of all transmitting nodes.

III. PROBLEM FORMULATION

This work considers a random medium access problem under concurrent transmissions originating from outer network clusters. Our goal is to maximize the network throughput of nodes, while minimizing the energy consumption caused by transmission. We use Shannon’s capacity as the measure of the network throughput, and a constant negative penalty as the transmission cost. We express our objective function as the net utility. The best-effort distributed transmission strategy maximizes the objective function.

We consider a Gaussian interference channel [21] between a transmitter and its home AP as described in Figure 1. When transmitter $i$ sends a packet to the AP $n$, the received signal $Y$ is given by

$$Y = \sqrt{h_i} \cdot X_i + \sum_{j \in X_i} \sqrt{\alpha_{mn} h_j} \cdot X_j + Z$$

where $h_i$ is the channel gain between transmitter $i$ and its home AP, $X_i$ is the transmitted signal from transmitter $i$, $X'$ is a set of the interfering nodes, $\alpha_{mn}$ is the crosstalk interference ratio between AP $m$ (which is $j$’s home AP) and AP $n$, and $Z$ is an Additive White Gaussian Noise (AWGN) signal. We assume that transmitter $i$ knows the channel gain between itself and its home AP, i.e., $h_i$. This can be reasonably achieved by considering a block fading channel where each user’s channel gain is invariant over a block of transmission. Also, we assume that each transmitter knows only the distribution of the interfering channel gains for other nodes rather than the exact values. The distribution of the interfering channel gains is given by $P(\alpha_{mn} h_j)$ where $h_j$ is the channel gain between interferer $j$ and the interferer’s home AP $m$, and $\alpha_{mn}$ is the crosstalk interference ratio between the interferer’s home AP $m$ and $h_j$’s home AP $n$. Since it is assumed that each node is much closer to its own home AP than to neighboring APs, the distance variation from a node to each different neighboring AP is negligible. Also, the crosstalk interference ratio is assumed to be symmetric. Therefore, we consider crosstalk interference ratio $\alpha_{mn} \approx E[\alpha_{mn}] := \alpha$, which is the average crosstalk interference ratio.

Our work is based on the SINR model, used in previous seminal works [22], [23]. The SINR model takes into account the received signal strength, the ambient noise level, and the interference from nodes concurrently transmitting. The model requires a minimum signal-to-noise ratio $\text{SNR}_{\text{th}}$, for a successful reception. We define the SNR $\gamma_i$ at the home
We formulate the Access Game as a one-shot, simultaneous-move game. To allow for a more realistic wireless environment, we consider that each channel is a Rayleigh fading channel where the probability distribution function (pdf) of the channel gain $h_i$ is given by $P(h_i) = \lambda \cdot e^{-\lambda h_i}$, where $1/\lambda$ is the average received power. Since each channel link is affected by many scatterers in each different environment, it is reasonable to assume that each channel is independently Rayleigh distributed. In this game, each transmitter knows its own channel gain $h_i$ from the feedback of a previous block transmission, and only the distribution of others’ channel gains $h_{-i}$. To describe the uncertainty of other users’ channels, the conditional distribution $P(h_{-i} | h_i)$ is defined, and called player $i$’s belief. In a Bayesian Nash Equilibrium, all players simultaneously maximize their own expected payoffs given their beliefs. In this static game setting, we define a strategy $S_i(\cdot)$ of player $i$ as a function of his type $h_i$.

**Definition 2** $S_i(h_i)$ is a Bayesian Nash Equilibrium if and only if:

$$S_i(h_i) \in \arg \max_{a_i \in S_i} \mathbb{E} [\Pi_i(a_i, S_{-i}(h_{-i})) | h_i]$$

for all $h_i$ and for all player $i$.

**Definition 3** In a Bayesian Nash Equilibrium, player $i$ will play Transmit if and only if

$$\mathbb{E} [\Pi_i(\text{Transmit}, S_{-i}(h_{-i})) | h_i] \geq \mathbb{E} [\Pi_i(\text{Backoff}, S_{-i}(h_{-i})) | h_i].$$

**Proposition 1** Any Bayesian Nash Equilibrium is given by a threshold strategy form as follows.

$$S_i(h_i) = \begin{cases} \text{Transmit} & \text{if } h_i \geq h_{th,i} \\ \text{Backoff} & \text{otherwise} \end{cases}$$

where $h_{th,i}$ is the transmission threshold of player $i$. 

**IV. CHANNEL ACCESS GAME**

We model nodes as selfish players that try to maximize their own payoff, comprising benefits (i.e., throughput) and costs (i.e., battery consumption). The nodes must decide on a strategy independently, assuming that the other nodes are also rational and self-interested. We apply game-theoretic concepts of equilibrium to identify the best tradeoff points between throughput and power consumption. We study a non-cooperative, simultaneous-move game where nodes cannot communicate or otherwise share information. Moreover, due to the fluctuating nature of the physical channel, our game is of incomplete information. 

**A. Modeling the Access Game**

In this section, we define the basic channel access game.

**Definition 1 (Static Channel Access Game)** We formulate an interference-aware channel access game as follows:

- **Players**: There are $N$ players, with each player $i \in \{1, 2, \ldots, N\}$ having type $h_i \geq 0$, representing its channel gain.
- **Actions**: $a_i \in \{\text{Transmit}, \text{Backoff}\}$ for all players $i = 1, \ldots, N$.

**V. STATIC GAME**

In this section, we consider a static channel access game where every player simultaneously picks a strategy to access a channel. This is a one-shot, simultaneous-move game. To allow for a more realistic wireless environment, we consider that each channel is a Rayleigh fading channel where the probability distribution function (pdf) of the channel gain $h_i$ is given by $P(h_i) = \lambda \cdot e^{-\lambda h_i}$, where $1/\lambda$ is the average received power. Since each channel link is affected by many scatterers in each different environment, it is reasonable to assume that each channel is independently Rayleigh distributed. In this game, each transmitter knows its own channel gain $h_i$ from the feedback of a previous block transmission, and only the distribution of others’ channel gains $h_{-i}$. To describe the uncertainty of other users’ channels, the conditional distribution $P(h_{-i} | h_i)$ is defined, and called player $i$’s belief. In a Bayesian Nash Equilibrium, all players simultaneously maximize their own expected payoffs given their beliefs. In this static game setting, we define a strategy $S_i(\cdot)$ of player $i$ as a function of his type $h_i$.

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- **Actions**: $a_i \in \{\text{Transmit}, \text{Backoff}\}$ for all players $i = 1, \ldots, N$.
Proof: When player $i$ chooses Transmit, the expected payoff is given by the following expression:

$$
E \left[ \Pi_i(\text{Transmit}, S_{-i}(h_{-i})) | h_i \right] = \sum_{x_i \cup x_i \cup (1, ..., N) - \{i\} \in X_i} P(S_j(h_j) = T) \cdot \prod_{j \in X_i} P(S_j(h_j) = B) \cdot P \left( \frac{h_i}{\alpha \sum_{j \in X_i} h_j + \sigma^2} \geq \text{SNR}_h \mid \{S_j(h_j) = T \, \forall j \in X_i\} \right) \cdot E \left( \log \left( 1 + \frac{h_i}{\alpha \sum_{j \in X_i} h_j + \sigma^2} \right) \mid \{S_j(h_j) = T \, \forall j \in X_i\}, \sum_{j \in X_i} h_j \leq \frac{1}{\alpha} (\text{SNR}_h - \sigma^2) \right) - \beta
$$

where $X_i = \{j \neq i : a_j = \text{Transmit}\}$, and $X_i = \{j \neq i : a_j = \text{Backoff}\}$.

The expected payoff function for Transmit is increasing in $h_i$ (see Appendix A). On the other hand, when player $i$ chooses Backoff, the expected payoff of player $i$ is 0. Any rational player will choose Transmit if and only if the expected payoff for Transmit is larger than the expected payoff for Backoff, i.e., $E \left[ \Pi_i(\text{Transmit}, S_{-i}(h_{-i})) | h_i \right] \geq 0$. This inequality is equivalent to transmitting whenever $h_i \geq h_{th,i}$. Therefore, any Bayesian Nash Equilibrium is given by a threshold strategy with threshold $h_{th,i}$ satisfying $E \left[ \Pi_i(\text{Transmit}, S_{-i}(h_{-i})) | h_{th,i} \right] = 0$.

Now this proposition allows us to focus only on a threshold-based channel access game. We search for a Bayesian Nash Equilibrium of threshold strategy form such that $S_i(h_i) = \text{Transmit}$ only if $h_i \geq h_{th,i}$, and otherwise Backoff for later channel access.

A. Static Game with Symmetric Bayesian Nash Equilibrium

We first consider the symmetric case where each user has the same transmission threshold, i.e., $h_{th,i} = h_{th}$, so that all players have the same payoff function. This means that every player’s optimal payoff would be the same, and thus every player achieves the same throughput in expectation, which guarantees equal channel share for all nodes. We formulate a symmetric channel access game, and derive a Bayesian Nash Equilibrium where the total number of active users is 2 and then generalize to $N > 2$.

1) With one interfering user: We first consider a simple case of interfering users where the total number of active users is $N = 2$. We denote by $F_c(x)$ the cumulative distribution function (cdf) of the exponential random variable, and $\bar{F}_c(x) = 1 - F_c(x)$.

Given $S_2(\cdot)$ and $h_1$, player 1 maximizes his expected payoff by playing a best response.

If player 1 plays Transmit,

$$
E \left[ \Pi_1(\text{Transmit}, S_2(h_2)) | h_1 \right] = P(h_2 < h_{th}) \left[ \log \left( \frac{1 + h_1}{\sigma^2} \right) - \beta \right]
$$

$$
+ P(h_2 \geq h_{th}) \left[ \frac{h_1}{\alpha h_2 + \sigma^2} \geq \text{SNR}_h \mid h_2 \geq h_{th} \right) \cdot E \left( \log \left( 1 + \frac{h_1}{\alpha h_2 + \sigma^2} \right) - \beta \mid h_{th} \leq h_2 \leq \frac{1}{\alpha} (\text{SNR}_h - \sigma^2) \right) + P \left( \frac{h_1}{\alpha h_2 + \sigma^2} < \text{SNR}_h \mid h_2 \geq h_{th} \right) (0 - \beta)
$$

$$
= F_c(h_{th}) \left[ \log \left( 1 + \frac{h_1}{\sigma^2} \right) - \beta \right]
$$

$$
+ \bar{F}_c(h_{th}) \left[ \frac{1}{\alpha} \left( \frac{h_1}{\text{SNR}_h} - \sigma^2 \right) - h_{th} \right]
$$

$$
- E \left( \log \left( 1 + \frac{h_1}{\alpha h_2 + \sigma^2} \right) \mid h_{th} \leq h_2 \leq \frac{1}{\alpha} (\text{SNR}_h - \sigma^2) \right)
$$

The above expected payoff function of player 1 is an increasing function in terms of $h_1$. Note that the memoryless property of exponential random variable is used in $(a)$.

If player 1 plays Backoff,

$$
E \left[ \Pi_1(\text{Backoff}, S_2(h_2)) | h_1 \right] = 0.
$$

Player 1 will play Transmit if and only if

$$
E \left[ \Pi_1(\text{Transmit}, S_2(h_2)) | h_1 \right] \geq E \left[ \Pi_1(\text{Backoff}, S_2(h_2)) | h_1 \right] = 0.
$$

$E \left[ \Pi_1(\text{Transmit}, S_2(h_2)) | h_1 \right]$ is an increasing function of $h_1$, so there exists a symmetric $h_{th}$ Bayesian Nash Equilibrium such that the above inequality holds where $h_1 \geq h_{th}$.

2) With multiple interfering users: We consider an $N$-player symmetric Bayesian channel access game. We formulate the channel access game for $N$ active users, and then derive a Bayesian Nash Equilibrium. We define a random variable which consists of the sum of $k$ independent and identically-distributed channel gains, i.e., $H_k = \sum_{i=1}^k h_i \sim \text{Gamma}(k, \frac{1}{k})$ where its pdf is given by $P(H_k = x) = \frac{1}{(k-1)!} x^{k-1} \cdot \frac{x^{k-2}}{\Gamma(k-1)}$. We denote $F_{g,k}(x)$, which is the cdf of Gamma random variable $H_k$, and $F_{g,k}(x|A)$ is its conditional cdf given $A$.

Given $S_{-i}(\cdot)$ and $h_i$, player $i$ maximizes its expected payoff by playing a best response.
If player $i$ plays Transmit,

$$E [Π_i (\text{Transmit}, S_{-i}(h_{-i})) | h_i]$$

$$= \sum_{k=0}^{N-1} (N-1)_k \cdot F_k(h_{th})^k F_k(h_{th})^{N-1-k}$$

$$\cdot \left[ P \left( \frac{h_i}{\alpha \sum_{m=1}^{k} h_m + \sigma^2} \geq \text{SNR}_{th} | A_k \right) \right]$$

$$\cdot \left[ F_{th,k} \left( \frac{h_i}{\alpha \sum_{m=1}^{k} h_m + \sigma^2} \right) \cdot \left( 1 + \frac{h_i}{\alpha \sum_{m=1}^{k} h_m + \sigma^2} \right) \right]$$

$$\cdot \left( \frac{1}{\alpha} \frac{h_i}{\text{SNR}_{th} - \sigma^2} - \beta \right)$$

$$+ \left( \frac{h_i}{\alpha \sum_{m=1}^{k} h_m + \sigma^2} < \text{SNR}_{th} | A_k \right) (-\beta)$$

$$= N \cdot (N-1) \cdot \sum_{k=0}^{N-1} (N-1)_k \cdot F_k(h_{th})^k F_k(h_{th})^{N-1-k}$$

$$\cdot \left[ F_{th,k} \left( \frac{h_i}{\alpha \sum_{m=1}^{k} h_m + \sigma^2} \right) \cdot \left( 1 + \frac{h_i}{\alpha \sum_{m=1}^{k} h_m + \sigma^2} \right) \right]$$

$$\cdot \left( \frac{1}{\alpha} \frac{h_i}{\text{SNR}_{th} - \sigma^2} - \beta \right)$$

where

$$A_k = \{ h_{j_1} \geq h_{th}, h_{j_2} \geq h_{th}, \ldots, h_{j_k} \geq h_{th}, \text{ and}$$

$$h_{j_k+1} < h_{th}, h_{j_k+2} < h_{th}, \ldots, h_{j_{N-1}} < h_{th} \}$$

If player $i$ plays Backoff,

$$E [Π_i (\text{Backoff}, S_{-i}(h_{-i})) | h_i] = 0.$$  

Accordingly, player $i$ will play Transmit if and only if

$$E [Π_i (\text{Transmit}, S_{-i}(h_{-i})) | h_i] \geq E [Π_i (\text{Backoff}, S_{-i}(h_{-i})) | h_i].$$

**Theorem 2** There exists a threshold $h_{th}$ such that $E [Π_i (\text{Transmit}, S_{-i}(h_{-i})) | h_i] \geq E [Π_i (\text{Backoff}, S_{-i}(h_{-i})) | h_i]$ if and only if $h_i \geq h_{th}$.

**Proof:** The expected payoff for Transmit of player $i$ can be reduced to the following by using a property

$$P(\sum_{j=1}^{k} h_j \leq \alpha) \geq h_{th} \geq \sum_{j=1}^{k} h_j \leq h_{th} \cdot \text{under} h_j \sim \exp(\lambda) \text{ with i.i.d.}$$

$$E [Π_i (\text{Transmit}, S_{-i}(h_{-i})) | h_i]$$

$$= \sum_{k=0}^{N-1} (N-1)_k \cdot F_k(h_{th})^k F_k(h_{th})^{N-1-k}$$

$$\cdot \left[ F_{th,k} \left( \frac{h_i}{\alpha \sum_{m=1}^{k} h_m + \sigma^2} \right) \cdot \left( 1 + \frac{h_i}{\alpha \sum_{m=1}^{k} h_m + \sigma^2} \right) \right]$$

$$\cdot \left( \frac{1}{\alpha} \frac{h_i}{\text{SNR}_{th} - \sigma^2} - \beta \right)$$

The above expected payoff function of player $i$ is an increasing function in terms of $h_i$, so there exists $h_{th}$ such that $E [Π_i (\text{Transmit}, S_{-i}(h_{-i})) | h_i] \geq 0$ holds when $h_i \geq h_{th}$. We can find the $h_{th}$ which satisfies $E [Π_i (\text{Transmit}, S_{-i}(h_{-i})) | h_{th}] = 0$ through the bisection method.

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**B. Static Game with non-Symmetric Bayesian Nash Equilibrium**

We generalize to Bayesian Nash Equilibria for non-symmetric threshold strategies. As in the symmetric case, the channel access game takes into account any concurrent interferers, which may now have different transmission thresholds.

If player $i$ plays Transmit,

$$E [Π_i (\text{Transmit}, S_{-i}(h_{-i})) | h_i]$$

$$= \sum_{x_i \cup \bar{x}_i = \{1, \ldots, N\} - \{i\} \in \bar{x}_i} \prod_{j \in \bar{x}_i} F_e(h_{th,j}) \cdot \prod_{j \in \bar{x}_i} F_e(h_{th,j})$$

$$\cdot \left[ P \left( \left( \frac{h_i}{\alpha \sum_{j \in \bar{x}_i} h_j + \sigma^2} \geq \text{SNR}_{th} \right) \text{ | } \{h_j \geq h_{th,j} \text{, } \forall j \in \bar{x}_i \} \right) \right]$$

$$\cdot \left[ E \left( \left( 1 + \frac{h_i}{\alpha \sum_{j \in \bar{x}_i} h_j + \sigma^2} \right) \cdot \left( \frac{1}{\alpha} \frac{h_i}{\text{SNR}_{th} - \sigma^2} - \beta \right) \right) \right]$$

where

$$\bar{x}_i = \{ j \neq i : a_j = \text{Transmit} \}, \text{ and } x_i = \{ j \neq i : a_j = \text{Backoff} \}.$$

If player $i$ plays Backoff,

$$E [Π_i (\text{Backoff}, S_{-i}(h_{-i})) | h_i] = 0.$$  

We find the thresholds $h_{th,i}$ for each player which satisfy

$$E [Π_i (\text{Transmit}, S_{-i}(h_{-i})) | h_{th,i}] = 0.$$  

**VI. EVALUATION OF STATIC GAME**

In this section, Bayesian Nash Equilibrium thresholds were computed by applying a Monte Carlo-based optimization scheme to solve the analytical formulation of the problem in Sec. V. As described in Sec. IV, we assume that the channel is a Rayleigh fading channel where the received power is exponentially distributed with parameter $\lambda$. For the rest of the evaluation section, unless otherwise noted, the results were obtained using $\lambda = 1$, $\alpha = 0.05$, $\sigma = 0.1$, $\text{SNR}_{th} = 10 \text{ dB}$, and $\beta = 1$.

To evaluate our game-theoretic MAC protocol, we first examine the stability of the Bayesian Nash Equilibrium in the case that a single node deviates from the equilibrium threshold strategy. This may happen if, for example, an adversary seizes
control of a node. Our payoff function models the actual utility that a node can extract from the system (Shannon’s capacity minus a transmission cost), so a rational user would deviate only if it increases the payoff.

Figure 2 shows that when the total number of users $N = 10$, if user 1 deviates from the BNE threshold of 1.49, the expected payoff starts to decrease. This confirms that each user does not have any incentive to deviate from the BNE point, guaranteeing the stability of the solution. If the threshold of the deviating user is greater than the BNE threshold, all other users achieve higher expected payoff because the deviating user transmits less frequently than other users, thereby reducing interference for other users.

Next, we compare the BNE threshold strategy with the globally optimal threshold strategy according to a global objective function, defined as the sum of the payoff functions of all $N$ nodes. We computed the globally optimal threshold values with a brute-force objective function computation of all possible threshold combinations (over a suitably fine discretization of the number line). Observe that, in practice, a globally optimal threshold strategy would be computed and distributed by a central server or AP, while on the other hand implementing the BNE threshold strategy is a decentralized algorithm that determines thresholds without any coordination. We define the efficiency ratio measure as the ratio of the global objective function of the Bayesian Nash Equilibrium solution and the globally optimal solution.

When $N = 10$, we computed the symmetric optimal threshold to be 2.13. The efficiency ratio for payoff is 0.58, while the efficiency ratio for throughput alone is 0.99. This means that the throughput of the BNE solution is nearly that of the symmetric optimal solution with some sacrifice of increased transmission cost. The extra cost comes from nodes transmitting more aggressively in order to prevent other users from deviating to an even lower threshold (which would hurt all those that do not deviate). Both the BNE threshold and the symmetric optimal threshold provide a balanced payoff, since all users have the same (i.e., symmetric) threshold value.

Figure 3 shows the instability at this globally optimal symmetric threshold strategy. Note that it is possible for a user to benefit from deviating from the optimal threshold. For example, if user 1 deviates by decreasing its threshold, its expected payoff increases due to the more frequent opportunity for transmission compared to other users. User 1 may continue to decrease his threshold down to 0, which provides the maximum payoff. But if all users also decrease their threshold too low, then all of the transmission trials would be unsuccessful, and thus every user would just consume transmission power without gaining throughput.

We also examine the general non-symmetric threshold strategy which achieves optimality in the global objective function. When $N = 10$, the optimal strategy is for one user to always transmit (i.e., a threshold of 0), while the other users always backoff. Here, the efficiency ratio for payoff is 0.52, and the efficiency ratio for throughput alone is 0.95. Similar to the symmetric case, the BNE solution yields a throughput fairly close to the throughput of the optimal solution with some sacrifice of increased cost due to transmitting more aggressively. Also, with the non-symmetric solution, only the transmitting user receives a positive payoff (an unbalanced allocation), and users have incentive to deviate, leading to instability. This result implies that any rational user will choose the symmetric BNE threshold strategy, which achieves both stability and balance among users.

Finally, we show how the threshold strategy of the Bayesian Nash Equilibrium should be adaptively tuned depending on the input parameters, i.e., the total number of active users $N$ over the networks, crosstalk interference ratio $\alpha$, AWGN noise power $\sigma^2$, hardware-dependent SNR threshold $\text{SNR}_{th}$, and the transmission cost $\beta$.

Figure 4 shows how different parameters affect the transmission probability. Observe throughout that, as expected, as the number of interfering users increases (and the amount of interference increases), each user increases its transmission threshold (thereby decreasing probability of transmission) in order to reduce the risk of transmission failure.

Figures 4(a) and 4(b) show the effects of the crosstalk interference ratio and environment noise power, respectively. As $\alpha$ and $\sigma$ increase, the transmission probability $P(h > h_{th})$ decreases. This means that each user would experience higher interference, which makes successful packet transmission much harder. Hence, each user would rationally decide to increase the transmission threshold so as to decrease useless battery consumption.

Figure 4(c) shows how different SNR thresholds impact the transmission probability. Usually, the SNR threshold depends on hardware. If a wireless receiver can support a relatively low SNR threshold under the same packet reception ratio, it would be reasonable to send packets more frequently according to a reduced transmission threshold.

Figure 4(d) shows that if the transmission cost increases, each user should increase the threshold accordingly because
or belief as to the distribution of other players’ channel gains $h_j$. First, we observe that the static game exhibits at least one pure Nash Equilibrium. We assume throughout this section that the channel gains $h_i$ of players are distinct, and indexed so that $h_1 < h_2 < \ldots < h_N$. This assumption is justified since having two equal channel gains is a 0 probability event.

In this section, for notational convenience, we simplify the arguments of the payoff function from Definition 1 as $\Pi_i(a_i, X_i)$, where $a_i$ is the action of player $i$ and $X_i = \{j \neq i : a_j = \text{Transmit}\}$ (i.e., we no longer explicitly include players that play Backoff in the payoff function).

**Proposition 3** The static game of Definition 1 exhibits at least one pure strategy NE, where there exists a threshold $h^*$ such that at equilibrium all players $i$ with $h_i \geq h^*$ play Transmit, while the rest play Backoff.

**Proof:** If $\Pi_N(\text{Transmit}, \emptyset) < 0$, then we set $h^* = h_N + \epsilon$ and we are done. Otherwise, player $N$ selects Transmit. If $\Pi_{N-1}(\text{Transmit}, \{N\}) < 0$, then we set $h^* = h_N$. Otherwise, player $N - 1$ selects Transmit; observe that player $N$ still has incentive to transmit, as $\Pi_N(\text{Transmit}, \{N-1\}) \geq 0$ since $\Pi$ is monotonically non-decreasing in channel gain and monotonically non-increasing in interference. We then iterate, stopping at the highest $j$ such that $\Pi_j(\text{Transmit}, \{j+1, j+2, \ldots, N\}) < 0$, and setting $h^* = h_{j+1}$. By the monotonicity of $\Pi$, no player $j' < j$ with $h_{j'} < h_j$ has incentive to transmit, while all players $j'' > j$ do have incentive to transmit since $\Pi_{j+1}(\text{Transmit}, \{j+2, j+3, \ldots, N\}) \geq 0$.

**A. Best-Response Dynamics**

In this section, we define the best response dynamics. The dynamics proceed in rounds, indexed by times $t \in \{0, 1, 2, 3, \ldots\}$. Denote by $a_i^t \in \{\text{Transmit}, \text{Backoff}\}$ be the action that player $i$ takes in round $t$, and $X_i^t = \{j \neq i : a_j^t = \text{Transmit}\}$. At round 0, each player $i$ initially plays $a_i^0 = \text{Backoff}$. For the duration of the dynamics, we assume that each player $i$’s type $h_i$ remains fixed. Note that this is consistent with our earlier assumption about block fading channels, where the channel statistics are fixed over constant-size blocks.

We now define our activation sequence, which specifies what player becomes activated in a given round $t$. In these dynamics, only the activated player may revise its action in round $t$, while all other players play according to their action in round $t - 1$.

**Definition 4 (Uniform Activation Sequence)** In the uniform activation sequence, for each round $t > 0$, a node is selected uniformly at random to be activated, denoted by $g(t)$.

For simplicity we have chosen the uniform distribution, but we remark that all of the results in this section also hold for any activation sequence in which nodes are selected according to any fixed distribution with full support. An alternative approach to the uniform activation sequence would be to model dynamics for this game according to a synchronous model in which all players make a best response simultaneously every round. However, best-response dynamics are not guaranteed to converge in the synchronous setting. Moreover, from an
implementation perspective, requiring all nodes to coordinate synchronous updates within the network is expensive, thereby negating the advantages that an otherwise lightweight and distributed procedure can bring.

We now define the natural myopic best-response dynamic for this game, and describe what it means for these dynamics to converge.

**Definition 5 (Best-response dynamics)** If \( g(t) = i \), then \( a_i^t = \text{Transmit} \iff \Pi_i(\text{Transmit}, X_{i-1}^{t-1}) \geq 0 \). That is, if player \( i \) is activated in round \( t \), then it transmits in round \( t \) if transmitting is a best response to the interference that \( i \) observed in round \( t-1 \), and otherwise backs off. For \( j \neq i \), \( a_j^t = a_j^{t-1} \).

Note that since \( \Pi_i \) depends only on the total interference and not on the identities of those transmitting, the dynamics only require that nodes observe last round’s total interference. Nodes never need to know the total number of players \( N \) or what other players’ gains are.

**Definition 6 (Convergence Criteria)** We say that the best-response dynamics in this game have converged by round \( t \) if \( \forall i, g(t) = i \implies a_i^t = a_i^{t-1} \). That is, the dynamics have converged by round \( t \) if every node’s best response would leave its action unchanged.

Notice that by the definition of Nash Equilibrium, if the dynamics have converged, then they do so to a NE of the game. We now proceed to show that the best-response dynamics do in fact converge, with probability 1, regardless of the initial channel gains \( h_i \) assigned to the players. That is, with probability 0, play ends up cycling indefinitely among a non-trivial set of strategy profiles.

Observe that while we will show that the best response dynamics always converge to a NE, they do not necessarily converge to the threshold-like equilibrium whose existence is guaranteed in Proposition 3. The existence of non-threshold equilibria depends on the particular realization of channel gains. We will prove this theorem by showing that for any time \( t \) at which the dynamics have not yet converged, we can construct an activation sequence of length \( \leq 2N \) after which the nodes will be playing according to a Nash Equilibrium of the game. Then, there is an activation sequence of length exactly \( N \) (cycling through all \( N \) players) which results in the convergence of the dynamics. For notational purposes, we denote by \( X^t = \{ i : a_i^t = \text{Transmit} \} \), and \( X_t = \{ j : a_j^t = \text{Backoff} \} \).

**Lemma 4** Suppose that at time \( t \), the best-response dynamics have not yet converged. Then there exists an activation sequence of length \( t_1 \leq N \) such that either (1): at time \( t + t_1 \), \( X_{t+t_1} = \{ 1, 2, 3, \ldots, k \} \) for some \( k \leq N \); that is, at time \( t + t_1 \) only and all players \( 1 \) through \( k \) play Transmit while the others play Backoff, or (2): by time \( t + t_1 \) play has reached a Nash Equilibrium.

**Proof:** At any time if the activation sequence that we are constructing leads to a NE, then we are done. Otherwise, suppose that at time \( t \) the strategy profile is not a NE of the game. Then there exists some player \( i \) whose best response is to change its action. If \( i \in X^t \), then by the monotonicity of the payoff function \( \Pi_i \) we know that \( i' = \min\{X^t \} \) also wants to change his action to Transmit (since if \( i' \neq i \), player \( i' \) only sees more interference, and has a lower channel gain, than player \( i \)). Then we would set \( t = t + 1 \) for the next element of the activation sequence. On the other hand, if \( i \in X^t \), then by the monotonicity of \( \Pi_i \), we know that \( i'' = \max\{X^t \} \) also wants to change his action to Transmit (since \( i'' \neq i \), player \( i'' \) would only see less interference while having a higher channel gain than player \( i \)). In this case we would set \( t = t'' \).

Repeating the above argument iteratively for times \( t + 1, t + 2, \ldots \) – that is, selecting for the next step in the activation sequence either the smallest member of \( X^t \) to switch to Backoff, or the largest member of \( X^t \) to switch to Transmit – then after \( t \leq N \) rounds we have the desired result.

**Lemma 5** Suppose that at time \( t \) play has not reached a Nash Equilibrium, but there exists \( k \leq N \) such that all players \( 1 \) through \( k \) play Backoff while the others play Transmit. Then there exists an activation sequence of length \( t_2 \leq N \) such that by round \( t + t_2 \) play has reached a Nash Equilibrium.

**Proof:** As play is not at a NE at time \( t \), then either \( \Pi_k(\text{Transmit}, \{ k + 1, k + 2, \ldots, N \} ) > 0 \) (i.e., \( k \) benefits by switching from Backoff to Transmit), or \( \Pi_{k+1}(\text{Transmit}, \{ k + 2, \ldots, N \} ) < 0 \) (i.e., \( k + 1 \) benefits by switching from Transmit to Backoff). If the former is true, then we set \( g(t+1) = k \), and then continue iteratively for players \( k - 1, k - 2, \ldots \) as in the algorithmic procedure of Proposition 3 until we find the player \( i \) with the largest gain \( h_i \) such that \( \Pi_i(\text{Transmit}, \{ i + 1, i + 2, \ldots, N \} ) < 0 \) and halt. If the latter is true, we simply reverse this procedure: we set \( g(t+1) = k + 1 \) and then continue iteratively for players \( k + 2, k + 3, \ldots \) until we find the player \( j \) with the smallest gain \( h_j \) such that \( \Pi_j(\text{Transmit}, \{ j + 1, j + 2, \ldots, N \} ) > 0 \).

**Theorem 6** The best-response dynamics with the uniform activation sequence converge with probability 1 to a pure strategy NE.

**Proof:** By Lemmas 4 and 5, at any given round \( t \) where the dynamics have not yet converged, there exists an activation sequence of length \( t_1 + t_2 \leq 2N \) that leads to a NE. This entire activation sequence of length \( \leq 2N \) occurs with probability \( \geq \frac{1}{N^2} \) (assuming uniform activation sequence). Therefore, the time to convergence is at least as fast as the time to the first success of a geometric random variable with success probability \( \geq \frac{1}{N^2} \), so the dynamics converge with probability 1.

While Theorem 6 proves that the dynamics converge with probability 1, in Figure 5 we show simulation results demonstrating the average time to convergence as a function of the number of players. Observe that convergence time appears to be polynomial in the number of players, suggesting relatively coarse bounds in the proof of Theorem 6.
By using the following inequalities,
\[
\begin{align*}
\int_{\hat{A}} \log \left( 1 + \frac{h_i + \epsilon}{\alpha \sum_{j \in X_i} h_j + \sigma^2} \right) \frac{P(h_{-i})}{dh_{-i}} \\
\geq \int_{A} \log \left( 1 + \frac{h_i + \epsilon}{\alpha \sum_{j \in X_i} h_j + \sigma^2} \right) \frac{P(h_{-i})}{dh_{-i}} \\
\geq \int_{A} \log \left( 1 + \frac{h_i}{\alpha \sum_{j \in X_i} h_j + \sigma^2} \right) \frac{P(h_{-i})}{dh_{-i}}
\end{align*}
\]
where \( \hat{A} = \{ h_{-i} | \sum_{j \in X_i} h_j \leq \frac{1}{\alpha} (\frac{h_i}{SNR_{\text{ideal}}} - \sigma^2) \} \), \( S_j(h_j) = T \) for \( \forall j \in X_i \), \( S_j(h_j) = B \) for \( \forall j \in X'_i \) and \( A \subseteq \hat{A} \), we complete the proof.

**REFERENCES**