Interference-Aware MAC Protocol for Wireless Networks by a Game-Theoretic Approach

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***Abstract*—We propose an interference-aware MAC protocol using a simple transmission strategy motivated by a game-theoretic approach. We formulate a channel access game, which considers nodes concurrently transmitting in nearby clusters, incorporating a realistic wireless communication model - the *SINR model*. Under *inter-cluster* interference, we derive a decentralized transmission strategy, which achieves a Bayesian Nash Equilibrium (BNE). The proposed MAC protocol balances network throughput and battery consumption at each transmission.**

We compare our BNE-based decentralized strategy with a centralized globally optimal strategy in terms of efficiency and balance. We further show that the transmission threshold should be adaptively tuned depending on the number of active users in the network, crosstalk, ambient noise, transmission cost, and radio-dependent receiver sensitivity. We also present a simple dynamic procedure for nodes to efficiently find a Nash Equilibrium (NE) without requiring each node to know the total number of active nodes or the channel gain distribution, and prove that this procedure is guaranteed to converge.

I. Introduction

Wireless networks have been widely deployed in the last two decades. Many access points (APs), Wi-Fi-enabled laptops, PDAs, and wireless sensors are deployed throughout offices, streets, campuses, and city environments. All wireless devices using 802.11x, 802.15.4 ZigBee, Bluetooth, and 802.16 WiMAX, share the ISM band in the 2.45 GHz range. Due to the shared spectrum, a wireless network affects other networks, and even small wireless devices can cause strong interference for other devices. Competitive channel usage causes additional volatility in wireless links, and can lead to critical performance degradation in terms of packet delivery [1], [2].

In traditional wireless 802.11x LAN networks, slotted ALOHA [3], CSMA (Carrier Sense Multiple Access) [4] and TDMA (Time Division Multiple Access) [5] have been used to mitigate the interference of wireless devices, and manage their medium access control (MAC) so that one user's channel access does not collide with another user's channel access. Because of their simplicity and robustness, these protocols have been widely used to manage mainly *intra-cluster* interference.

Recent experimental studies [6], [7] show that the *inter-cluster* interference in 802.11 can be a dominant cause of throughput degradation. As the usage of Wi-Fi networks rapidly increases, nearby access points must compete to access the same channels, generally without any coordination or guiding central authority. Accordingly, network throughput is reduced due to increased packet collisions.

To mitigate this effect, several decentralized algorithms motivated by game theory have been proposed [8]–[15]. In centralized approaches, the network system cannot easily protect from user deviation (i.e., a unilateral bid to increase throughput), and so may result in an unbalanced outcome [16]. On the other hand, the Nash Equilibria found in these decentralized approaches characterize points where a selfish user cannot benefit by unilaterally deviating; such an approach is resilient to deviation and ensures the robustness of the network system. However, these works do not explicitly handle the effect of interference, and are unable to accurately model realistic channel environments as they do not consider concurrent transmissions. Some recent game-theoretic works [17], [18] do consider the effect of interference and obtain a complex NE-based power adaptation scheme to mitigate packet collision.

The key advantage of studying MAC protocol design from a game theoretic perspective is that, by turning nodes into selfish players, an otherwise complex system can reach efficient outcomes in a lightweight and distributed manner, while reducing the communication overhead between nodes and their home APs, and between neighboring APs. In this paper, we focus on the channel access problem under *inter-cluster* interference from nearby APs, and find a simple transmission strategy that determines whether to `Transmit` or `Backoff` depending on channel conditions. We focus on the case that each node experiences instantaneous signal-to-noise ratio (SNR) fluctuation caused by interferers concurrently transmitting. First, assuming that each transmitter knows the number of active users at the next transmission, its own channel gain, and the probability distribution of the gains of interfering channels, we derive a threshold-based transmission scheme balancing benefit (i.e., throughput) and cost (i.e., transmission cost) which achieves a Bayesian Nash Equilibrium. Then, we present a simple dynamic procedure for nodes to efficiently find a Nash Equilibrium without requiring nodes to know the total number of active nodes or even their channel gain distributions.

Our proposed MAC protocol does not assign any backoff slot beforehand for later access. Rather, after a random backoff time, given a time-varying channel state, a subsequent channel access is attempted. Our protocol is not restricted to a specific homogeneous network, and can be extended to heterogeneous networks such as 802.11x, 802.15.4 sensor networks and Bluetooth, by using each different crosstalk ratio.

To the best of our knowledge, this paper is the first to formulate a channel access game under the effect of inter-cluster interference incorporating a realistic wireless communication model. Our main contributions can be summarized as:

- We formulate an interference-aware channel access game, which considers concurrent transmission of nodes from different APs, incorporating random fading channels and a realistic wireless communication model.
- We prove the existence of a Bayesian Nash Equilibrium of the game, and compute it using numerical methods.
- We compare our BNE-based decentralized strategy with a centralized globally optimal strategy in terms of efficiency and balance.
- We present a simple dynamic procedure for nodes to efficiently find a Nash Equilibrium without requiring each node to know the total number of active nodes or the channel gain distribution, and prove that this procedure is guaranteed to converge.

The rest of this paper is organized as follows: In Sec. II, we discuss related work. Sec. III formally defines our channel access problem. In Sec. IV, we motivate our use of game theory, and then formulate the problem as a channel access game. Sec. V describes the proposed static game-theoretic approach, and then we evaluate the approach in Sec. VI. We prove a convergence result using best-response dynamics to find a Nash Equilibrium in Sec. VII, and then conclude in Sec. VIII.

II. Related Work

The channel access game problem has recently been studied in the wireless MAC research community, with game-theoretic MAC schemes taking their solutions from three different categories: NE-based backoff time adaptation, NE-based power control, and NE-based transmission schemes that depend on channel conditions.

Some previous works [19], [20] consider the exponential backoff scheme in the IEEE 802.11 standard, and formulate the backoff protocol as a non-cooperative game where each link tries to maximize its own utility. They prove the existence of a Nash Equilibrium, and show that the NE-based backoff scheme converges to a globally optimal point of network utility.

Adaptive power control schemes are proposed in [17], [18]. Both works formulate the problem as a power control game, where strategies are given by interference-dependent transmission power profiles. These works explicitly consider the interference effect as in our work, but their strategies provide schemes that optimizes over power adaptation, whereas our work optimizes a transmission scheme given a constant power.

Some recent works are closely related to our approach. Qin and Berry [9] propose a channel-aware ALOHA protocol, and compute a user's transmission probability given its own channel gain. They assume that a collision occurs when two or more users transmit packets in the same slot. In [12], [15], transmission strategies decide whether or not to access the channel, i.e., `Transmit` or `Backoff`, depending on the SNR. They assume that each user's SNR is an independent and identically-distributed (i.i.d.) random variable, and follows the probability density function of a Rayleigh fading channel. However, when we consider nodes concurrently transmitting out of the home AP, i.e., taking into account the inter-cluster interference effect, their assumptions are no longer valid. This is because the SNR is not in fact i.i.d., but depends on the number of nodes concurrently transmitting, which cannot be known before the game ends. Our work relaxes these assumptions so that only each channel gain is assumed to be i.i.d., and the SNR is expressed in terms of the channel gains of all transmitting nodes.

III. Problem Formulation

This work considers a random medium access problem under concurrent transmissions originating from outer network clusters. Our goal is to maximize the network throughput of nodes, while minimizing the energy consumption caused by transmission. We use Shannon's capacity as the measure of the network throughput, and a constant negative penalty as the transmission cost. We express our objective function as the net utility. The best-effort distributed transmission strategy maximizes the objective function.

We consider a Gaussian interference channel [21] between a transmitter and its home AP as described in Figure 1. When transmitter i sends a packet to the AP n, the received signal Y is given by

$$Y = \sqrt{h_i} \cdot X_i + \sum_{j \in \mathcal{X}} \sqrt{\alpha_{mn} h_j} \cdot X_j + Z$$

where h_i is the channel gain between transmitter i and its home AP, X_i is the transmitted signal from transmitter i, $\mathcal{X}$ is a set of the interfering nodes, α_{mn} is the crosstalk interference ratio between AP m (which is j's home AP) and AP n, and Z is an Additive White Gaussian Noise (AWGN) signal. We assume that transmitter i knows the channel gain between itself and its home AP, i.e., h_i. This can be reasonably achieved by considering a block fading channel where each user's channel gain is invariant over a block of transmission. Also, we assume that each transmitter knows only the distribution of the interfering channel gains for other nodes rather than the exact values. The distribution of the interfering channel gains is given by $P(\alpha_{mn} h_j)$ where h_j is the channel gain between interferer j and the interferer's home AP m, and α_{mn} is the crosstalk interference ratio between the interferer's home AP m and h_i's home AP n. Since it is assumed that each node is much closer to its own home AP than to neighboring APs, the distance variation from a node to each different neighboring AP is negligible. Also, the crosstalk interference ratio is assumed to be symmetric. Therefore, we consider crosstalk interference ratio $\alpha_{mn} \approx \mathbb{E}[\alpha_{mn}] := \alpha$, which is the average crosstalk interference ratio.

Our work is based on the *SINR model*, used in previous seminal works [22], [23]. The SINR model takes into account the received signal strength, the ambient noise level, and the interference from nodes concurrently transmitting. The model requires a minimum signal-to-noise ratio SNR_{th}, for a successful reception. We define the SNR γ_i at the home

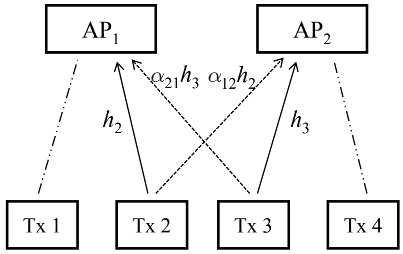


Fig. 1. Interference channel model where h_i is the channel gain between transmitter i and its home AP, and α_{mn} is the crosstalk interference ratio between AP m and AP n

AP from node i, and consider that successful packet delivery occurs only when the following inequality holds:

$$\gamma_i = \frac{h_i p_i}{\alpha \sum_{j \in \mathcal{X}} h_j p_j + \sigma^2} \geq \mathrm{SNR_{th}}$$

where p_i is the power used by the transmitter of link i, σ^2 is an AWGN noise power, $\alpha(< 1)$ is the crosstalk interference ratio, and $\mathcal{X}$ is the set of all the interfering nodes. We use a fixed power in the MAC protocol, thereby allowing us to have $p_i = p$ and assigning the unity power 1 for mathematical convenience.

According to the model, each user's strategy of whether or not to transmit a packet depends on the number of users and their channel status under interference, crosstalk interference ratio, AWGN noise power, and the minimum SNR threshold (which represents the receiver's radio sensitivity). The random medium access problem can then be described as finding a transmission scheme, i.e., a strategy $S_i = \texttt{Transmit}$ only if *a proposed transmission condition* holds.

IV. Channel Access Game

We model nodes as selfish players that try to maximize their own payoff, comprising benefits (i.e., throughput) and costs (i.e., battery consumption). The nodes must decide on a strategy independently, assuming that the other nodes are also rational and self-interested. We apply game-theoretic concepts of equilibrium to identify the best tradeoff points between throughput and power consumption. We study a non-cooperative, simultaneous-move game where nodes cannot communicate or otherwise share information. Moreover, due to the fluctuating nature of the physical channel, our game is one of incomplete information.

A. Modeling the Access Game

In this section, we define the basic channel access game.

Definition 1 (Static Channel Access Game) *We formulate an interference-aware channel access game as follows:*

- *Players: There are N players, with each player $i \in \{1, 2, \ldots, N\}$ having type $h_i \geq 0$, representing its channel gain.*
- *Actions: $a_i \in \{\texttt{Transmit}, \texttt{Backoff}\}$ for all players $i = 1, \ldots, N$*
- *Payoff function:*

$$\Pi_i(\texttt{Backoff}, \mathbf{a_{-i}}) = 0,$$

$$\Pi_i(\texttt{Transmit}, \mathbf{a_{-i}}) = r_i(\mathbf{a_{-i}}) - \beta$$

where β is the cost of consuming transmission power, $\mathbf{a_{-i}}$ is the vector of actions of players other than i, and $r_i(\mathbf{a_{-i}})$ is the network throughput expressed by

$$r_i(\mathbf{a_{-i}}) = \begin{cases} \log(1 + \frac{h_i}{\alpha \sum_{j \in \mathcal{X}_i} h_j + \sigma^2}) & \textit{if} \frac{h_i}{\alpha \sum_{j \in \mathcal{X}_i} h_j + \sigma^2} \geq \textit{SNR}_{th} \\ 0 & \textit{otherwise} \end{cases}$$

where

$$\mathcal{X}_i = \{j \neq i : a_j = \texttt{Transmit}\}.$$

V. Static Game

In this section, we consider a static channel access game where every player simultaneously picks a strategy to access a channel. This is a one-shot, simultaneous-move game. To allow for a more realistic wireless environment, we consider that each channel is a Rayleigh fading channel where the probability distribution function (pdf) of the channel gain h_i is given by $P(h_i) = \lambda \cdot e^{-\lambda h_i}$ where $1/\lambda$ is the average received power. Since each channel link is affected by many scatterers in each different environment, it is reasonable to assume that each channel is independently Rayleigh distributed. In this game, each transmitter knows its own channel gain h_i from the feedback of a previous block transmission, and only the distribution of others' channel gains $\mathbf{h_{-i}}$. To describe the uncertainty of other users' channels, the conditional distribution $P(\mathbf{h_{-i}}|h_i)$ is defined, and called player i's *belief*. In a Bayesian Nash Equilibrium, all players simultaneously maximize their own expected payoffs given their beliefs. In this static game setting, we define a strategy $S_i(\cdot)$ of player i as a function of his type h_i.

Definition 2 *$S_i(h_i)$ is a Bayesian Nash Equilibrium if and only if:*

$$S_i(h_i) \in \arg \max_{a_i \in S_i} \mathbb{E}\left[\Pi_i(a_i, \mathbf{S_{-i}}(\mathbf{h_{-i}}))|h_i\right]$$

for all h_i and for all player i.

Definition 3 *In a Bayesian Nash Equilibrium, player i will play* Transmit *if and only if*

$$\mathbb{E}\left[\Pi_i(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}}))|h_i\right] \geq \mathbb{E}\left[\Pi_i\left(\texttt{Backoff}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_i\right].$$

Proposition 1 *Any Bayesian Nash Equilibrium is given by a threshold strategy form as follows.*

$$S_i(h_i) = \begin{cases} \texttt{Transmit} & \textit{if } h_i \geq h_{th,i} \\ \texttt{Backoff} & \textit{otherwise} \end{cases}$$

where $h_{th,i}$ is the transmission threshold of player i.

Proof: When player i chooses $\texttt{Transmit}$, the expected payoff is given by the following expression:

$$\begin{aligned}
&\mathbb{E}\left[\Pi_i(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}}))|h_i\right] \\
=& \sum_{\mathcal{X}_i \bigcup \bar{\mathcal{X}}_i = \{1,\dots,N\}-\{i\}} \prod_{j\in\mathcal{X}_i} P\left(S_j(h_j)=\texttt{T}\right)\cdot \prod_{j\in\bar{\mathcal{X}}_i} P\left(S_j(h_j)=\texttt{B}\right) \\
&\cdot P\left(\frac{h_i}{\alpha\sum_{j\in\mathcal{X}_i} h_j+\sigma^2} \geq \text{SNR}_{\text{th}} \mid \{S_j(h_j)=\texttt{T}\ ,\ \forall j\in\mathcal{X}_i\}\right) \\
&\cdot \mathbb{E}\Big(\log\big(1+\frac{h_i}{\alpha\sum_{j\in\mathcal{X}_i} h_j+\sigma^2}\big) \mid \{S_j(h_j)=\texttt{T}\ ,\ \forall j\in\mathcal{X}_i\}, \\
&\qquad \sum_{j\in\mathcal{X}_i} h_j \leq \frac{1}{\alpha}\big(\frac{h_i}{\text{SNR}_{\text{th}}}-\sigma^2\big)\Big) \quad -\beta
\end{aligned}$$

where $\mathcal{X}_i = \{j\neq i : a_j = \texttt{Transmit}\}$, and $\bar{\mathcal{X}}_i = \{j\neq i : a_j = \texttt{Backoff}\}$.

The expected payoff function for $\texttt{Transmit}$ is increasing in h_i (see Appendix A). On the other hand, when player i chooses $\texttt{Backoff}$, the expected payoff of player i is 0. Any rational player will choose $\texttt{Transmit}$ if and only if the expected payoff for $\texttt{Transmit}$ is larger than the expected payoff for $\texttt{Backoff}$, i.e., $\mathbb{E}\left[\Pi_i(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}}))|h_i\right] \geq 0$. This inequality is equivalent to transmitting whenever $h_i \geq h_{th,i}$. Therefore, any Bayesian Nash Equilibrium is given by a threshold strategy with threshold $h_{th,i}$ satisfying $\mathbb{E}\left[\Pi_i\left(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_{th,i}\right] = 0$. ■

Now this proposition allows us to focus only on a threshold-based channel access game. We search for a Bayesian Nash Equilibrium of threshold strategy form such that $S_i(h_i) = \texttt{Transmit}$ only if $h_i \geq h_{th,i}$, and otherwise $\texttt{Backoff}$ for later channel access.

A. Static Game with Symmetric Bayesian Nash Equilibrium

We first consider the symmetric case where each user has the same transmission threshold, i.e., $h_{th,i} = h_{th}$, so that all players have the same payoff function. This means that every player's optimal payoff would be the same, and thus every player achieves the same throughput in expectation, which guarantees equal channel share for all nodes. We formulate a symmetric channel access game, and derive a Bayesian Nash Equilibrium where the total number of active users is 2 and then generalize to $N > 2$.

1) With one interfering user: We first consider a simple case of interfering users where the total number of active users is $N = 2$. We denote by $F_e(x)$ the cumulative distribution function (cdf) of the exponential random variable, and $\bar{F}_e(x) = 1 - F_e(x)$.

Given $S_2(\cdot)$ and h_1, player 1 maximizes his expected payoff by playing a best response.

If player 1 plays $\texttt{Transmit}$,

$$\begin{aligned}
&\mathbb{E}\left[\Pi_1\left(\texttt{Transmit}, S_2(h_2)\right)|h_1\right] \\
=\ & P(h_2 < h_{th})\left[\log\left(1+\frac{h_1}{\sigma^2}\right)-\beta\right] \\
+\ & P(h_2 \geq h_{th})\Big[P\left(\frac{h_1}{\alpha h_2+\sigma^2} \geq \text{SNR}_{\text{th}} \mid h_2 \geq h_{th}\right) \\
&\cdot \mathbb{E}\left(\log\left(1+\frac{h_1}{\alpha h_2+\sigma^2}\right)-\beta \mid h_{th} \leq h_2 \leq \frac{1}{\alpha}\big(\frac{h_1}{\text{SNR}_{\text{th}}}-\sigma^2\big)\right) \\
&+P\left(\frac{h_1}{\alpha h_2+\sigma^2} < \text{SNR}_{\text{th}} \mid h_2 \geq h_{th}\right)(0-\beta)\Big] \\
\overset{(a)}{=}\ & F_e(h_{th})\left[\log\left(1+\frac{h_1}{\sigma^2}\right)-\beta\right] \\
+\ & \bar{F}_e(h_{th})\Big[F_e\left(\frac{1}{\alpha}\left(\frac{h_1}{\text{SNR}_{\text{th}}}-\sigma^2\right)-h_{th}\right) \\
&\cdot \mathbb{E}\left(\log\left(1+\frac{h_1}{\alpha h_2+\sigma^2}\right)-\beta \mid h_{th} \leq h_2 \leq \frac{1}{\alpha}\big(\frac{h_1}{\text{SNR}_{\text{th}}}-\sigma^2\big)\right) \\
&+\bar{F}_e\left(\frac{1}{\alpha}\left(\frac{h_1}{\text{SNR}_{\text{th}}}-\sigma^2\right)-h_{th}\right)(0-\beta)\Big] \\
=\ & F_e(h_{th})\left[\log\left(1+\frac{h_1}{\sigma^2}\right)-\beta\right] \\
+\ & \bar{F}_e(h_{th})\Big[F_e\left(\frac{1}{\alpha}\left(\frac{h_1}{\text{SNR}_{\text{th}}}-\sigma^2\right)-h_{th}\right) \\
&\cdot \mathbb{E}\left(\log\left(1+\frac{h_1}{\alpha h_2+\sigma^2}\right) \mid h_{th} \leq h_2 \leq \frac{1}{\alpha}\big(\frac{h_1}{\text{SNR}_{\text{th}}}-\sigma^2\big)\right) \\
&-\beta\Big].
\end{aligned}$$

The above expected payoff function of player 1 is an increasing function in terms of h_1. Note that the memoryless property of exponential random variable is used in (a).

If player 1 plays $\texttt{Backoff}$,

$$\mathbb{E}\left[\Pi_1\left(\texttt{Backoff}, S_2(h_2)\right)|h_1\right] = 0.$$

Player 1 will play $\texttt{Transmit}$ if and only if

$$\begin{aligned}
&\mathbb{E}\left[\Pi_1(\texttt{Transmit}, S_2(h_2))|h_1\right] \\
&\qquad\qquad \geq\ \mathbb{E}\left[\Pi_1(\texttt{Backoff}, S_2(h_2))|h_1\right].
\end{aligned}$$

$\mathbb{E}\left[\Pi_1\left(\texttt{Transmit}, S_2(h_2)\right)|h_1\right]$ is an increasing function of h_1, so there exists a symmetric h_{th} Bayesian Nash Equilibrium such that the above inequality holds where $h_1 \geq h_{th}$.

2) With multiple interfering users: We consider an N-player symmetric Bayesian channel access game. We formulate the channel access game for N active users, and then derive a Bayesian Nash Equilibrium. We define a random variable which consists of the sum of k independent and identically-distributed channel gains,
i.e., $H_k = \sum_{i=1}^{k} h_i \sim \text{Gamma}(k, \frac{1}{\lambda})$ where its pdf is given by $P(H_k = x) = \frac{\lambda e^{-\lambda x}(\lambda x)^{k-1}}{(k-1)!}$. We denote $F_{g,k}(x)$, which is the cdf of Gamma random variable H_k, and $F_{g,k}(x|A)$ is its conditional cdf given A.

Given $\mathbf{S_{-i}}(\cdot)$ and h_i, player i maximizes its expected payoff by playing a best response.

If player i plays Transmit,

$$
\begin{aligned}
& \mathbb{E}\left[\Pi_i\left(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_i\right] \\
= & \sum_{k=0}^{N-1}\binom{N-1}{k}\bar{F}_e(h_{th})^k F_e(h_{th})^{N-1-k} \\
\cdot & \Big[P\left(\frac{h_i}{\alpha\sum_{m=1}^{k}h_{j_m}+\sigma^2}\geq \text{SNR}_{\text{th}} \mid A_k\right) \\
& \cdot\mathbb{E}\Big(\log\left(1+\frac{h_i}{\alpha\sum_{m=1}^{k}h_{j_m}+\sigma^2}\right)-\beta \mid A_k, \\
& \sum_{m=1}^{k}h_{j_m}\leq\frac{1}{\alpha}\big(\frac{h_i}{\text{SNR}_{\text{th}}}-\sigma^2\big)\Big) \\
& +P\left(\frac{h_i}{\alpha\sum_{m=1}^{k}h_{j_m}+\sigma^2}<\text{SNR}_{\text{th}} \mid A_k\right)(-\beta)\Big] \\
= & \sum_{k=0}^{N-1}\binom{N-1}{k}\bar{F}_e(h_{th})^k F_e(h_{th})^{N-1-k} \\
& \Big[F_{g,k}\left(\frac{1}{\alpha}\left(\frac{h_i}{\text{SNR}_{th}}-\sigma^2\right) \mid A_k\right) \\
& \cdot\mathbb{E}\left(\log\left(1+\frac{h_i}{\alpha H_k+\sigma^2}\right) \mid A_k, H_k\leq\frac{1}{\alpha}\big(\frac{h_i}{\text{SNR}_{\text{th}}}-\sigma^2\big)\right)-\beta\Big]
\end{aligned}
$$

where

$$
\begin{aligned}
A_k = \quad \{ \quad & h_{j_1}\geq h_{th}, h_{j_2}\geq h_{th},\ldots,h_{j_k}\geq h_{th}, \text{ and} \\
& h_{j_{k+1}}<h_{th}, h_{j_{k+2}}<h_{th},\ldots,h_{j_{N-1}}<h_{th} \quad \}.
\end{aligned}
$$

If player i plays Backoff,

$$\mathbb{E}\left[\Pi_i\left(\texttt{Backoff}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_i\right]=0.$$

Accordingly, player i will play Transmit if and only if

$$
\begin{aligned}
& \mathbb{E}\left[\Pi_i(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}}))|h_i\right] \\
& \qquad\qquad \geq \mathbb{E}\left[\Pi_i\left(\texttt{Backoff}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_i\right].
\end{aligned}
$$

Theorem 2 *There exists a threshold h_{th} such that $\mathbb{E}\left[\Pi_i\left(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_i\right] \geq \mathbb{E}\left[\Pi_i\left(\texttt{Backoff}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_i\right]$ if and only if $h_i\geq h_{th}$.*

Proof: The expected payoff for Transmit of player i can be reduced to the following by using a property $P(\sum_{j=1}^{k}h_j\leq a \mid h_j\geq h_{th} \text{ for } 1\leq j\leq k) = P(\sum_{j=1}^{k}h_j\leq a-k\cdot h_{th})$ under $h_j\sim\exp(\lambda)$ with i.i.d.:

$$
\begin{aligned}
& \mathbb{E}\left[\Pi_i\left(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_i\right] \\
= & \sum_{k=0}^{N-1}\binom{N-1}{k}\bar{F}_e(h_{th})^k F_e(h_{th})^{N-1-k} \\
& \Big[F_{g,k}\left(\frac{1}{\alpha}\left(\frac{h_i}{\text{SNR}_{th}}-\sigma^2\right)-k\cdot h_{th}\right) \\
& \cdot\mathbb{E}\left(\log\left(1+\frac{h_i}{\alpha H_k+\sigma^2}\right) \mid A_k, H_k\leq\frac{1}{\alpha}\big(\frac{h_i}{\text{SNR}_{\text{th}}}-\sigma^2\big)\right)-\beta\Big]
\end{aligned}
$$

The above expected payoff function of player i is an increasing function in terms of h_i, so there exists h_{th} such that $\mathbb{E}\left[\Pi_i\left(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_i\right]\geq 0$ holds when $h_i\geq h_{th}$. We can find the h_{th} which satisfies $\mathbb{E}\left[\Pi_i\left(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_{th}\right]=0$ through the bisection method. ∎

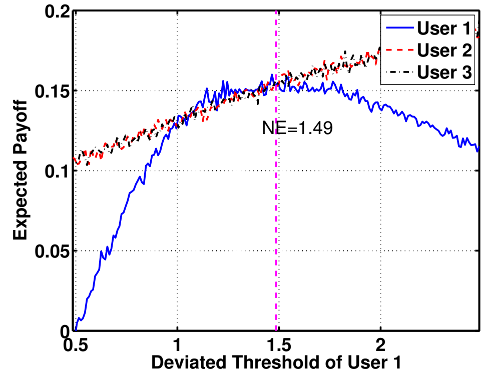


Fig. 2. Expected payoff depending on the threshold of a deviating user while all the other users keep the BNE threshold where $N=10$. If user 1 deviates from the BNE threshold, his expected payoff decreases. Users 4,…,10 show the same patterns as users 2 and 3, so they are omitted.

B. Static Game with non-Symmetric Bayesian Nash Equilibrium

We generalize to Bayesian Nash Equilibria for non-symmetric threshold strategies. As in the symmetric case, the channel access game takes into account any concurrent interferers, which may now have different transmission thresholds.

If player i plays Transmit,

$$
\begin{aligned}
& \mathbb{E}\left[\Pi_i(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}}))|h_i\right] \\
= & \sum_{\mathcal{X}_i\cup\bar{\mathcal{X}}_i=\{1,\ldots,N\}-\{i\}}\prod_{j\in\mathcal{X}_i}\bar{F}_e(h_{th,j})\cdot\prod_{j\in\bar{\mathcal{X}}_i}F_e(h_{th,j}) \\
& \cdot P\left(\frac{h_i}{\alpha\sum_{j\in\mathcal{X}_i}h_j+\sigma^2}\geq\text{SNR}_{\text{th}} \mid \{h_j\geq h_{th,j}\ ,\ \forall j\in\mathcal{X}_i\}\right) \\
& \cdot\mathbb{E}\Big(\log\big(1+\frac{h_i}{\alpha\sum_{j\in\mathcal{X}_i}h_j+\sigma^2}\big) \mid \{h_j\geq h_{th,j}\ ,\ \forall j\in\mathcal{X}_i\}, \\
& \quad \sum_{j\in\mathcal{X}_i}h_j\leq\frac{1}{\alpha}\big(\frac{h_i}{\text{SNR}_{\text{th}}}-\sigma^2\big)\Big) \quad -\beta
\end{aligned}
$$

where $\mathcal{X}_i=\{j\neq i: a_j=\texttt{Transmit}\}$, and $\bar{\mathcal{X}}_i=\{j\neq i: a_j=\texttt{Backoff}\}$.

If player i plays Backoff,

$$\mathbb{E}\left[\Pi_i\left(\texttt{Backoff}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_i\right]=0.$$

We find the thresholds $h_{th,i}$ for each player which satisfy $\mathbb{E}\left[\Pi_i\left(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}})\right)|h_{th,i}\right]=0$.

VI. Evaluation of Static Game

In this section, Bayesian Nash Equilibrium thresholds were computed by applying a Monte Carlo-based optimization scheme to solve the analytical formulation of the problem in Sec. V. As described in Sec. IV, we assume that the channel is a Rayleigh fading channel where the received power is exponentially distributed with parameter λ. For the rest of the evaluation section, unless otherwise noted, the results were obtained using $\lambda=1$, $\alpha=0.05$, $\sigma=0.1$, $\text{SNR}_{\text{th}}=10$ dB, and $\beta=1$.

To evaluate our game-theoretic MAC protocol, we first examine the stability of the Bayesian Nash Equilibrium in the case that a single node deviates from the equilibrium threshold strategy. This may happen if, for example, an adversary seizes control of a node. Our payoff function models the actual utility that a node can extract from the system (Shannon's capacity minus a transmission cost), so a rational user would deviate only if it increases the payoff.

Figure 2 shows that when the total number of users $N = 10$, if user 1 deviates from the BNE threshold of 1.49, the expected payoff starts to decrease. This confirms that each user does not have any incentive to deviate from the BNE point, guaranteeing the stability of the solution. If the threshold of the deviating user is greater than the BNE threshold, all other users achieve higher expected payoff because the deviating user transmits less frequently than other users, thereby reducing interference for other users.

Next, we compare the BNE threshold strategy with the globally optimal threshold strategy according to a global objective function, defined as the sum of the payoff functions of all N nodes. We computed the globally optimal threshold values with a brute-force objective function computation of all possible threshold combinations (over a suitably fine discretization of the number line). Observe that, in practice, a globally optimal threshold strategy would be computed and distributed by a central server or AP, while on the other hand implementing the BNE threshold strategy is a decentralized algorithm that determines thresholds without any coordination. We define the efficiency ratio measure as the ratio of the global objective function of the Bayesian Nash Equilibrium solution and the globally optimal solution.

When $N = 10$, we computed the symmetric optimal threshold to be 2.13. The efficiency ratio for payoff is 0.58, while the efficiency ratio for throughput alone is 0.99. This means that the throughput of the BNE solution is nearly that of the symmetric optimal solution with some sacrifice of increased transmission cost. The extra cost comes from nodes transmitting more aggressively in order to prevent other users from deviating to an even lower threshold (which would hurt all those that do not deviate). Both the BNE threshold and the symmetric optimal threshold provide a balanced payoff, since all users have the same (i.e., symmetric) threshold value.

Figure 3 shows the instability at this globally optimal symmetric threshold strategy. Note that it is possible for a user to benefit from deviating from the optimal threshold. For example, if user 1 deviates by decreasing its threshold, its expected payoff increases due to the more frequent opportunity for transmission compared to other users. User 1 may continue to decrease his threshold down to 0, which provides the maximum payoff. But if all users also decrease their threshold too low, then all of the transmission trials would be unsuccessful, and thus every user would just consume transmission power without gaining throughput.

We also examine the general non-symmetric threshold strategy which achieves optimality in the global objective function. When $N = 10$, the optimal strategy is for one user to always `Transmit` (i.e., a threshold of 0), while the other users always `Backoff`. Here, the efficiency ratio for payoff is 0.52, and the efficiency ratio for throughput alone is 0.95. Similar to the symmetric case, the BNE solution yields a throughput

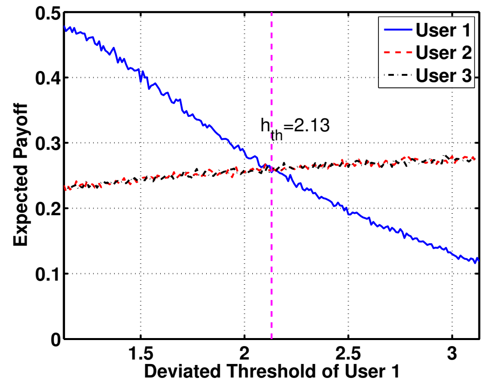


Fig. 3. Expected payoff depending on the threshold of a deviating user while all other users keep the designated non-BNE threshold where $N = 10$. If selfish user 1 deviates from the non-BNE threshold to a smaller threshold, its expected payoff increases. Any selfish user would deviate from the designated threshold by decreasing the threshold down to 0, and just consume transmission power without successful packet delivery. Users 4,. . . ,10 show the same patterns as users 2 and 3, so they are omitted.

fairly close to the throughput of the optimal solution with some sacrifice of increased cost due to transmitting more aggressively. Also, with the non-symmetric solution, only the transmitting user receives a positive payoff (an unbalanced allocation), and users have incentive to deviate, leading to instability. This result implies that any rational user will choose the symmetric BNE threshold strategy, which achieves both stability and balance among users.

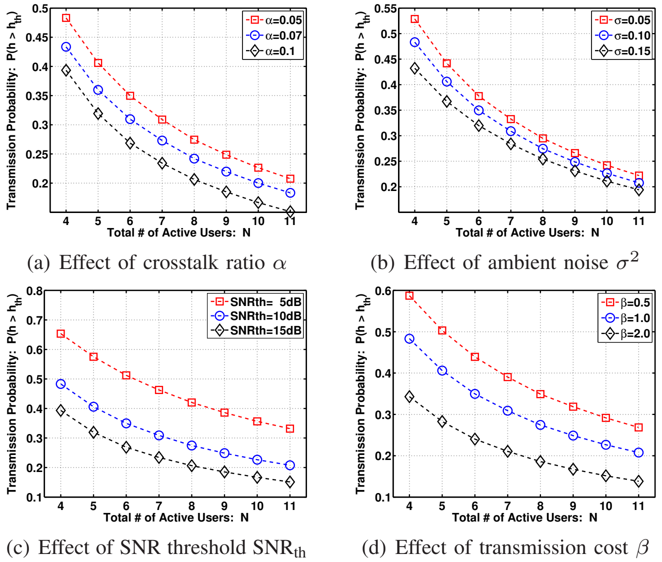
Finally, we show how the threshold strategy of the Bayesian Nash Equilibrium should be adaptively tuned depending on the input parameters, i.e., the total number of active users N over the networks, crosstalk interference ratio α, AWGN noise power σ^2, hardware-dependent SNR threshold $\mathrm{SNR}_{\mathrm{th}}$, and the transmission cost β.

Figure 4 shows how different parameters affect the transmission probability. Observe throughout that, as expected, as the number of interfering users increases (and the amount of interference increases), each user increases its transmission threshold (thereby decreasing probability of transmission) in order to reduce the risk of transmission failure.

Figures 4(a) and 4(b) show the effects of the crosstalk interference ratio and environment noise power, respectively. As α and σ increase, the transmission probability $P(h > h_{th})$ decreases. This means that each user would experience higher interference, which makes successful packet transmission much harder. Hence, each user would rationally decide to increase the transmission threshold so as to decrease useless battery consumption.

Figure 4(c) shows how different SNR thresholds impact the transmission probability. Usually, the SNR threshold depends on hardware. If a wireless receiver can support a relatively low SNR threshold under the same packet reception ratio, it would be reasonable to send packets more frequently according to a reduced transmission threshold.

Figure 4(d) shows that if the transmission cost increases, each user should increase the threshold accordingly because



(a) Effect of crosstalk ratio α (b) Effect of ambient noise σ^2

(c) Effect of SNR threshold $\mathrm{SNR_{th}}$ (d) Effect of transmission cost β

Fig. 4. Transmission probability depending on parameters. As N, α, σ, $\mathrm{SNR_{th}}$, β increase, the transmission probability decreases.

any rational user would like to reduce the risk of packet loss. This implies that under more expensive transmission costs, each user should transmit a packet more sparingly and only when its channel is relatively good enough for packet delivery in order to reduce losses.

VII. Dynamic Game

The Bayesian Nash Equilibrium solution concept that we have so far discussed is a static solution concept, in that it describes a fixed point of the system where all nodes are simultaneously playing a best response to other nodes' actions. While such a solution may be stable and efficient, it is not always clear how the nodes will *find* this equilibrium in the first place. In Sec. V, we demonstrated a Bayesian Nash Equilibrium, but this approach requires each node to independently compute the threshold corresponding to the equilibrium. This computation requires a non-trivial fixed-point calculation, which may be expensive. Moreover, this approach requires nodes to know the total number of nodes N in the channel access game, achieved by having the APs count the number of the next scheduled transmitters and then each home AP inform its respective nodes of the number N.

But the key motivation for studying MAC protocol design from a game theoretic perspective is that, by turning nodes into selfish players, an otherwise complex system can reach efficient outcomes in a lightweight and distributed manner. With each node behaving according to its own best interest, expensive coordination between nodes and communication with a central processor can hopefully be reduced.

In this section, we propose a simple dynamic procedure for the nodes to cheaply and easily *find* an equilibrium of the static game of Definition 1. By applying a best-response dynamic, we show that node actions converge to a threshold-like Nash Equilibrium, without ever requiring a node to know the total number N of players or even have a knowledge of or belief as to the distribution of other players' channel gains h_j. First, we observe that the static game exhibits at least one pure Nash Equilibrium. We assume throughout this section that the channel gains h_i of players are distinct, and indexed so that $h_1 < h_2 < \ldots < h_N$. This assumption is justified since having two equal channel gains is a 0 probability event.

In this section, for notational convenience, we simplify the arguments of the payoff function from Definition 1 as $\Pi_i(a_i, \mathcal{X}_i)$, where a_i is the action of player i and $\mathcal{X}_i = \{j \neq i : a_j = \texttt{Transmit}\}$ (i.e., we no longer explicitly include players that play `Backoff` in the payoff function).

Proposition 3 *The static game of Definition 1 exhibits at least one pure strategy NE, where there exists a threshold h^* such that at equilibrium all players i with $h_i \geq h^*$ play* `Transmit`, *while the rest play* `Backoff`.

Proof: If $\Pi_N(\texttt{Transmit}, \emptyset) < 0$, then we set $h^* = h_N + \epsilon$ and we are done. Otherwise, player N selects `Transmit`. If $\Pi_{N-1}(\texttt{Transmit}, \{N\}) < 0$, then we set $h^* = h_N$. Otherwise, player $N-1$ selects `Transmit`; observe that player N still has incentive to transmit, as $\Pi_N(\texttt{Transmit}, \{N-1\}) \geq 0$ since Π is monotonically non-decreasing in channel gain and monotonically non-increasing in interference. We then iterate, stopping at the highest j such that $\Pi_j(\texttt{Transmit}, \{j+1, j+2, \ldots, N\}) < 0$, and setting $h^* = h_{j+1}$. By the monotonicity of Π, no player $j' < j$ with $h_{j'} < h_j$ has incentive to transmit, while all players $j'' > j$ do have incentive to transmit since $\Pi_{j+1}(\texttt{Transmit}, \{j+2, j+3, \ldots, N\}) \geq 0$. ■

A. Best-Response Dynamics

In this section, we define the best response dynamics. The dynamics proceed in rounds, indexed by times $t \in \{0, 1, 2, 3, ...\}$. Denote by $a_i^t \in \{\texttt{Transmit}, \texttt{Backoff}\}$ to be the action that player i takes in round t, and $\mathcal{X}_i^t = \{j \neq i : a_j^t = \texttt{Transmit}\}$. At round 0, each player i initially plays $a_i^0 = \texttt{Backoff}$. For the duration of the dynamics, we assume that each player i's type h_i remains fixed. Note that this is consistent with our earlier assumption about block fading channels, where the channel statistics are fixed over constant-size blocks.

We now define our activation sequence, which specifies what player becomes *activated* in a given round t. In these dynamics, only the activated player may revise its action in round t, while all other players play according to their action in round $t-1$.

Definition 4 (Uniform Activation Sequence) *In the uniform activation sequence, for each round $t > 0$, a node is selected uniformly at random to be activated, denoted by $g(t)$.*

For simplicity we have chosen the uniform distribution, but we remark that all of the results in this section also hold for any activation sequence in which nodes are selected according to any fixed distribution with full support. An alternative approach to the uniform activation sequence would be to model dynamics for this game according to a *synchronous* model in which all players make a best response simultaneously every round. However, best-response dynamics are not guaranteed to converge in the synchronous setting. Moreover, from an implementation perspective, requiring all nodes to coordinate synchronous updates within the network is expensive, thereby negating the advantages that an otherwise lightweight and distributed procedure can bring.

We now define the natural myopic best-response dynamic for this game, and describe what it means for these dynamics to converge.

Definition 5 (Best-response dynamics) *If $g(t) = i$, then $a_i^t = \texttt{Transmit} \iff \Pi_i(\texttt{Transmit}, \mathcal{X}_i^{t-1}) \geq 0$. That is, if player i is activated in round t, then it transmits in round t if transmitting is a best response to the interference that i observed in round $t-1$, and otherwise backs off. For $j \neq i$, $a_j^t = a_j^{t-1}$.*

Note that since Π_i depends only on the total interference and not on the identities of those transmitting, the dynamics only require that nodes observe last round's total interference. Nodes never need to know the total number of players N or what other players' gains are.

Definition 6 (Convergence Criteria) *We say that the best-response dynamics in this game have* converged *by round t if $\forall i,\ g(t) = i \implies a_i^t = a_i^{t-1}$. That is, the dynamics have converged by round t if* every *node's best response would leave its action unchanged.*

Notice that by the definition of Nash Equilibrium, if the dynamics have converged, then they do so to a NE of the game. We now proceed to show that the best-response dynamics do in fact converge, with probability 1, regardless of the initial channel gains h_i assigned to the players. That is, with probability 0, play ends up cycling indefinitely among a non-trivial set of strategy profiles.

Observe that while we will show that the best response dynamics always converge to a NE, they do not necessarily converge to the threshold-like equilibrium whose existence is guaranteed in Proposition 3. The existence of non-threshold equilibria depends on the particular realization of channel gains. We will prove this theorem by showing that for any time t at which the dynamics have not yet converged, we can construct an activation sequence of length $\leq 2N$ after which the nodes will be playing according to a Nash Equilibrium of the game. Then, there is an activation sequence of length exactly N (cycling through all N players) which results in the convergence of the dynamics. For notational purposes, we denote by $\mathcal{X}^t = \{i : a_i^t = \texttt{Transmit}\}$, and $\bar{\mathcal{X}}^t = \{j : a_j^t = \texttt{Backoff}\}$.

Lemma 4 *Suppose that at time t, the best-response dynamics have not yet converged. Then there exists an activation sequence of length $t_1 \leq N$ such that either **(1)**: at time $t + t_1$, $\bar{\mathcal{X}}^{t+t_1} = \{1, 2, 3, \ldots, k\}$ for some $k \leq N$; that is, at time $t + t_1$ only and all players 1 through k play* `Backoff` *while the others play* `Transmit`*, or **(2)**: by time $t + t_1$ play has reached a Nash Equilibrium.*

Proof: At any time if the activation sequence that we are constructing leads to a NE, then we are done. Otherwise, suppose that at time t the strategy profile is *not* a NE of the game. Then there exists some player i whose best response is to change its action. If $i \in \mathcal{X}^t$, then by the monotonicity of the payoff function Π_i we know that $i' = \min\{\mathcal{X}^t\}$ also wants to change his action to `Backoff` (since if $i' \neq i$, player i' only sees more interference, and has a lower channel gain, than player i). Then we would set $g(t+1) = i'$ for the next element of the activation sequence. On the other hand, if $i \in \bar{\mathcal{X}}^t$, then by the monotonicity of Π_i we know that $i'' = \max\{\bar{\mathcal{X}}^t\}$ also wants to change his action to `Transmit` (since if $i'' \neq i$, player i'' would only see less interference while having a higher channel gain than player i). In this case we would set $g(t+1) = i''$.

Repeating the above argument iteratively for times $t+1, t+2, \ldots$ – that is, selecting for the next step in the activation sequence either the smallest member of $\mathcal{X}^t$ to switch to `Backoff`, or the largest member of $\bar{\mathcal{X}}^t$ to switch to `Transmit` – then after $t_1 \leq N$ rounds we have the desired result. ■

Lemma 5 *Suppose that at time t play has not reached a Nash Equilibrium, but there exists $k \leq N$ such that all players 1 through k play* `Backoff` *while the others play* `Transmit`*. Then there exists an activation sequence of length $t_2 \leq N$ such that by round $t + t_2$ play has reached a Nash Equilibrium.*

Proof: As play is not at a NE at time t, then either $\Pi_k(\texttt{Transmit}, \{k+1, k+2, \ldots, N\}) \geq 0$ (i.e., k benefits by switching from `Backoff` to `Transmit`), or $\Pi_{k+1}(\texttt{Transmit}, \{k+2, \ldots, N\}) < 0$ (i.e., $k+1$ benefits by switching from `Transmit` to `Backoff`). If the former is true, then we set $g(t+1) = k$, and then continue iteratively for players $k-1, k-2, \ldots$ as in the algorithmic procedure of Proposition 3 until we find the player i with the *largest* gain h_i such that $\Pi_i(\texttt{Transmit}, \{i+1, i+2, \ldots, N\}) < 0$ and halt. If the latter is true, we simply reverse this procedure: we set $g(t+1) = k+1$ and then continue iteratively for players $k+2, k+3, \ldots$ until we find the player j with the *smallest* gain h_j such that $\Pi_j(\texttt{Transmit}, \{j+1, j+2, \ldots, N\}) \geq 0$. ■

Theorem 6 *The best-response dynamics with the uniform activation sequence converge with probability 1 to a pure strategy NE.*

Proof: By Lemmas 4 and 5, at *any* given round t where the dynamics have not yet converged, there exists an activation sequence of length $t_1 + t_2 \leq 2N$ that leads to a NE. This entire activation sequence of length $\leq 2N$ occurs with probability $\geq \frac{1}{N^{2N}}$ (assuming uniform activation sequence). Therefore, the time to convergence is at least as fast as the time to the first success of a geometric random variable with success probability $\geq \frac{1}{N^{2N}}$, so the dynamics converge with probability 1. ■

While Theorem 6 proves that the dynamics converge with probability 1, in Figure 5 we show simulation results demonstrating the average *time* to convergence as a function of the number of players. Observe that convergence time appears to be polynomial in the number of players, suggesting relatively coarse bounds in the proof of Theorem 6.

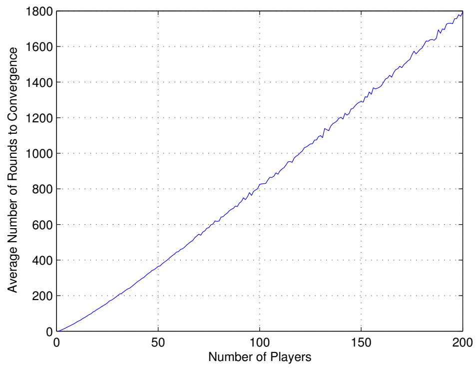


Fig. 5. Average number of rounds until best-response dynamics converge. The average is taken over randomness in Rayleigh fading channels and over randomness in uniform activation sequence. Parameters used are as in Sec. VI.

VIII. Conclusion

We have presented a novel interference-aware game-theoretic MAC protocol, both in the static and dynamic game setting. In the static setting, our proposed strategy is a Bayesian Nash Equilibrium, where nodes choose to transmit according to a channel gain-based threshold, balancing the benefit of throughput with the cost of battery consumption. While the BNE strategy provides a good approximation to the globally optimal solution, it is most valuable in that it ensures a *stable* outcome where nodes have no incentive to deviate from the symmetric equilibrium. We show how the threshold strategy should be adaptively changed depending on the number of interfering users, crosstalk interference ratio, ambient noise level, transmission cost, and radio-dependent SNR sensitivity. In the dynamic setting, we propose a natural and practical best-response dynamic procedure for nodes to cheaply and easily find an equilibrium of the static game.

For future work, we may relax the assumption that the crosstalk interference ratio is approximated by the average value. Considering statistical variations of the crosstalk would provide a more accurate and realistic interference model.

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Appendix A

Proposition 7 $\mathbb{E}\left[\Pi_i(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}}))|h_i\right]$ *is an increasing function of* h_i.

Proof: We show that the expected function value of $h_i+\epsilon$ where $\epsilon > 0$ is greater than or equal to that of h_i in its integral form as follows:

$$\begin{aligned}&\mathbb{E}\left[\Pi_i(\texttt{Transmit}, \mathbf{S_{-i}}(\mathbf{h_{-i}}))|h_i\right]\\&= \sum_{\mathcal{X}_i \cup \bar{\mathcal{X}}_i = \{1,\ldots,N\}-\{i\}} \int_A \log\big(1+\frac{h_i}{\alpha\sum_{j\in\mathcal{X}_i} h_j + \sigma^2}\big)P(\mathbf{h_{-i}})\mathbf{dh_{-i}} - \beta\end{aligned}$$

where $A = \{\ \mathbf{h_{-i}}\ |\ \sum_{j\in\mathcal{X}_i} h_j \le \frac{1}{\alpha}\big(\frac{h_i}{\mathrm{SNR_{th}}} - \sigma^2\big),\ S_j(h_j) = \texttt{T}$ for $\forall j \in \mathcal{X}_i,\ S_j(h_j) = \texttt{B}$ for $\forall j \in \bar{\mathcal{X}}_i\ \}$.

By using the following inequalities,

$$\begin{aligned}&\int_{\tilde{A}} \log\big(1+\frac{h_i+\epsilon}{\alpha\sum_{j\in\mathcal{X}_i} h_j + \sigma^2}\big)P(\mathbf{h_{-i}})\mathbf{dh_{-i}}\\&\ge \int_{A} \log\big(1+\frac{h_i+\epsilon}{\alpha\sum_{j\in\mathcal{X}_i} h_j + \sigma^2}\big)P(\mathbf{h_{-i}})\mathbf{dh_{-i}}\\&\ge \int_{A} \log\big(1+\frac{h_i}{\alpha\sum_{j\in\mathcal{X}_i} h_j + \sigma^2}\big)P(\mathbf{h_{-i}})\mathbf{dh_{-i}}\end{aligned}$$

where $\tilde{A} = \{\ \mathbf{h_{-i}}\ |\ \sum_{j\in\mathcal{X}_i} h_j \le \frac{1}{\alpha}\big(\frac{h_i+\epsilon}{\mathrm{SNR_{th}}} - \sigma^2\big),\ S_j(h_j) = \texttt{T}$ for $\forall j \in \mathcal{X}_i,\ S_j(h_j) = \texttt{B}$ for $\forall j \in \bar{\mathcal{X}}_i\ \}$ and $A \subset \tilde{A}$, we complete the proof. ■

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